

ON THE ADMISSIBILITY OF REPLACING THE HEAT CONDUCTION EQUATION OF A PIPE WALL BY THE HEAT BALANCE EQUATION IN INVESTIGATING TRANSIENTS IN HEAT EXCHANGERS

V. V. Krashennnikov

UDC 536.21:66.045.1

An expression is obtained to determine the fraction of the thermal resistance of a pipe wall which must be referred to the inner boundary when using the heat balance equation for the wall instead of the heat conduction equation (for a heat exchanger with independent heating). Limits of applicability of the model with temperature concentrated along the pipe radius are given.

Widely used in the analysis of transients in heat exchangers is the simplification that the heat conduction equation of the pipe wall can be replaced by the heat balance equation. To take account of the heat conduction of the metal, a certain fraction of the wall thermal resistance is referred to the inner (or outer) boundary, i.e., heat-exchange coefficients with a correction for the resistance of the metal heat conduction are introduced [1]. Up to now, individual computations of particular cases by exact and approximate methods are known on whose basis it is impossible to make general deductions.

The possibility of using a simplified model to determine the frequency characteristics of a tubular heat exchanger with independent heating (internal heat liberation in the wall or heating from outside by radiation, say) for a heat carrier with changing properties (near-critical state, boiling fluid) is analyzed herein by comparing models with the pipe wall temperature distributed and concentrated along the radius.

Let us first examine the model with wall temperature distributed along the radius. The processes occurring in a tubular heat exchanger under the standard assumptions can be described by a system of equations including the energy, stream continuity and motion equations (in a one-dimensional approximation), and the wall heat conduction equation:

$$\frac{1}{v} \cdot \frac{\partial I}{\partial t} + G \frac{\partial I}{\partial l} = \frac{4\alpha_2}{d_2} (\Phi_{r_2} - \Theta) + \frac{\partial P}{\partial t} - Gv \frac{\partial G}{\partial t}; \quad (1)$$

$$\frac{\partial G}{\partial l} = \frac{1}{v^2} \cdot \frac{\partial v}{\partial t}; \quad (2)$$

$$\frac{\partial P}{\partial l} = -\xi \frac{G^2 v}{2d_2} - \frac{9.8}{v} \sin \gamma - \frac{G}{v} \cdot \frac{\partial v}{\partial t} - \frac{\partial G}{\partial t}; \quad (3)$$

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial \Phi}{\partial r} + \frac{Q_{in}}{\lambda_M} = \frac{1}{a} \cdot \frac{\partial \Phi}{\partial t}; \quad (4)$$

for  $r = r_2$

$$\frac{\partial \Phi}{\partial r} = \frac{\alpha_2}{\lambda_M} (\Phi_{r_2} - \Theta). \quad (5)$$

For  $r = r_1$

$$\frac{\partial \Phi}{\partial r} = 0 \text{ for internal heat liberation,} \quad (6)$$

F. E. Dzerzhinskii All-Union Thermal Engineering Institute, Moscow. Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 19, No. 6, pp. 1079-1087, December, 1970. Original article submitted July 25, 1969.

© 1973 Consultants Bureau, a division of Plenum Publishing Corporation, 227 West 17th Street, New York, N. Y. 10011. All rights reserved. This article cannot be reproduced for any purpose whatsoever without permission of the publisher. A copy of this article is available from the publisher for \$15.00.

or

$$\frac{\partial \Phi}{\partial r} = \frac{Q_1}{\lambda_M} \text{ for external heating} \quad (7)$$

(both cases will be examined simultaneously later).

Let us investigate the process for small sinusoidal perturbations of the enthalpy, discharge, and pressure of the medium at the inlet, and also for perturbations by the heat flux, i.e., let us assume that

$$\begin{aligned} I_{\text{out}} &= I_{\text{out}0} + i e^{j\omega t}; & G_{\text{out}} &= G_0 (1 + g_{\text{out}} e^{j\omega t}); & P_{\text{out}} &= P_{\text{out}0} + p_{\text{out}} e^{j\omega t}; \\ Q_{\text{in}} &= Q_{\text{in}0} (1 + q e^{j\omega t}); & Q_1 &= Q_{10} (1 + q e^{j\omega t}). \end{aligned} \quad (8)$$

Because of these perturbations changes in the parameters along the heat exchanger will occur. In a linear approximation we can assume

$$\begin{aligned} I &= I_0(l) + i(l) e^{j\omega t}, & G &= G_0 [1 + g(l) e^{j\omega t}], \\ P &= P_0(l) + p(l) e^{j\omega t}, & \Phi &= \Phi_0(l, r) + \psi(l, r) e^{j\omega t}. \end{aligned} \quad (9)$$

Deviations in the temperature of the medium and the specific volume are determined by the expressions

$$\begin{aligned} \Theta &= \Theta_0 + \left( \frac{i}{c_p} + \delta p \right) e^{j\omega t}, \\ v &= v_0 + (\sigma_I i + \sigma_p p) e^{j\omega t}. \end{aligned} \quad (10)$$

Here we used the notation

$$\delta = \left( \frac{\partial \Theta}{\partial P} \right)_I, \quad \sigma_I = \left( \frac{\partial v}{\partial I} \right)_p, \quad \sigma_p = \left( \frac{\partial v}{\partial P} \right)_I.$$

Considering the changes in the parameters to occur relatively slowly, we can assume that the coefficient of hydraulic resistance does not vary in a nonstationary process, and the coefficient of heat exchange varies according to stationary regularities. Taking account of the dependence of  $\alpha_2$  on the enthalpy, discharge, and heat flux (in the boiling case), we will have

$$\alpha_2 = \alpha_{20} \left\{ 1 + \frac{1}{\mu_q} \left[ \frac{1}{\alpha_{20}} \cdot \frac{\partial \alpha_2}{\partial I} i + \mu_g g + \frac{\partial \alpha_2}{\partial Q_2} \left( \psi_{r_2} - \frac{i}{c_p} - \delta p \right) \right] e^{j\omega t} \right\}, \quad (11)$$

where the coefficients are

$$\mu_q = 1 - \frac{Q_{20}}{\alpha_{20}} \left( \frac{\partial \alpha_2}{\partial Q_2} \right); \quad \mu_g = \frac{G_0}{\alpha_{20}} \left( \frac{\partial \alpha_2}{\partial G} \right).$$

Let us substitute (9)-(11) into (1)-(7). Let us eliminate the statics equation and, for convenience, let us replace the length coordinate  $l$  by the enthalpy  $I_0$  in the static mode [ $dl = (d_2 G_0 / 4 Q_{20}) dI_0$ ]. Using the notation  $\mu_I = [1 - (c_p Q_{20} / \alpha_{20}^2)] (\partial \alpha_2 / \partial I)$ ,  $T_p = d_2 / \xi G_0 v_0$ ,  $\xi = d_1 / d_2$ , we obtain a system of equations for the deviations:

$$\frac{di}{dI_0} = \frac{\alpha_{20}}{Q_{20} \mu_q} \psi_{r_2} + \frac{1}{\mu_q} \left[ \mu_g g - \frac{\alpha_{20} \delta}{Q_{20}} p - \frac{\alpha_{20} \mu_I}{Q_{20} c_p} i \right] - \frac{j\omega d_2}{4 Q_{20} v_0} i - \left[ 1 + \frac{j\omega G_0^2 v_0 d_2}{4 Q_{20}} \right] g + \frac{j\omega d_2}{4 Q_{20}} p; \quad (12)$$

$$\frac{dg}{dI_0} = - \frac{j\omega d_2}{4 Q_{20} v_0^2} (\sigma_I i + \sigma_p p); \quad (13)$$

$$\frac{dp}{dI_0} = - \frac{\xi G_0^3}{4 Q_{20}} \left[ \frac{1}{2} - \frac{9.8 d_2 \sin \gamma}{\xi G_0^3 v_0^2} + j\omega T_p \right] (\sigma_I i + \sigma_p p) - \frac{\xi G_0^3 v_0}{4 Q_{20}} (1 + j\omega T_p) g; \quad (14)$$

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial \psi}{\partial r} - j \frac{\omega}{a} \psi = - \frac{q Q_{\text{in}0}}{\lambda_M}. \quad (15)$$

The boundary conditions will be:

for  $r = r_2$

$$\lambda_M \frac{\partial \psi}{\partial r} = \frac{\alpha_{20}}{\mu_q} \psi_{r_2} + \frac{Q_{20}}{\mu_q} \left( \mu_g g - \frac{\alpha_{20}}{Q_{20}} \delta p - \frac{\alpha_{20} \mu_l}{Q_{20} c_p} i \right); \quad (16)$$

for  $r = r_1$

$$\frac{\partial \psi}{\partial r} = 0; \quad (17)$$

or

$$\lambda_M \frac{\partial \psi}{\partial r} = q \frac{Q_{20}}{\xi}; \quad (18)$$

and for  $l = 0$

$$i = i_{\text{out}}, g = g_{\text{out}}, p = p_{\text{out}} \quad (19)$$

Solving [15] taking account of the boundary conditions (16) and (17) or (16) and (18) (for  $Q_{\text{in}0} = 0$ ), respectively [2], we find for the wall temperature deviation on the inner boundary of the pipe:

$$\psi_{r_2} = \frac{Q_{20}}{\alpha_{20}} \cdot \frac{\text{Bi}}{(\mu_q K - \text{Bi})} \left[ \mu_g g - \frac{\alpha_{20} \delta}{Q_{20}} p - \frac{\mu_l \alpha_{20}}{Q_{20} c_p} i \right] - \frac{Q_{20}}{\alpha_{20}} \cdot \frac{2jK \text{Bi} \mu_q}{(\mu_q K - \text{Bi}) y_2^2 (\xi^2 - 1)} q \quad (20)$$

in the case of internal heat liberation and

$$\psi_{r_2} = \frac{Q_{20}}{\alpha_{20}} \cdot \frac{\text{Bi}}{(\mu_q K - \text{Bi})} \left[ \mu_g g - \frac{\alpha_{20} \delta}{Q_{20}} p - \frac{\mu_l \alpha_{20}}{Q_{20} c_p} i \right] + \frac{Q_{20}}{\alpha_{20}} \cdot \frac{\text{Bi} \mu_q}{(\mu_q K - \text{Bi}) \nabla \xi y_2} q \quad (21)$$

for heating from outside. Here

$$y_1 = r_1 \sqrt{\frac{\omega}{a}}, \quad y_2 = r_2 \sqrt{\frac{\omega}{a}}, \quad \text{Bi} = \frac{\alpha_{20} r_2}{\lambda_M};$$

$$\nabla = [(\text{ker}' y_1 + j \text{kei}' y_1)(\text{ber} y_2 + j \text{bei} y_2) - (\text{ber}' y_1 + j \text{bei}' y_1)(\text{ker} y_2 + j \text{kei} y_2)];$$

$$K = \frac{y_2}{\nabla} [(\text{ker}' y_1 + j \text{kei}' y_1)(\text{ber}' y_2 + j \text{bei}' y_2) - (\text{ber}' y_1 + j \text{bei}' y_1)(\text{ker}' y_2 + j \text{kei}' y_2)].$$

Substituting  $\psi_{r_2}$  into (12), we obtain

$$\begin{aligned} \frac{di}{dI_0} = & - \left[ \frac{j\omega d_2}{4Q_{20} v_0} + \frac{\alpha_{20}}{Q_{20} c_p} \cdot \frac{\mu_l K}{(\mu_l K - \text{Bi})} \right] i - \left[ 1 - \frac{\mu_g K}{\mu_q K - \text{Bi}} \right. \\ & \left. + \frac{j\omega G_0^2 v_0 d_2}{4Q_{20}} \right] g - \left[ \frac{\alpha_{20} \delta K}{Q_{20} (\mu_q K - \text{Bi})} - \frac{j\omega d_2}{4Q_{20}} \right] p + Fq, \end{aligned} \quad (22)$$

where F has the value

$$F = - \frac{2jK \text{Bi}}{y_2^2 (\xi^2 - 1) (\mu_q K - \text{Bi})} \quad (23)$$

for internal heat liberation, and

$$F = \frac{\text{Bi}}{\nabla \xi y_2 (\mu_q K - \text{Bi})} \quad (24)$$

for heating from outside.

Therefore, in the general case transients are described by the system (22), (13), and (14) with the boundary conditions (19) for small perturbations in heat exchangers with uniform heating along the length (internal heat sources or heating from outside). This system can be used to compute the dynamic characteristics of heat exchangers with a single-phase heat carrier in the near-critical state [3, 4]. As a particular case ( $c_p \rightarrow \infty$ ,  $\mu_g = 0$ ,  $\mu_q \approx 0.3$ ), we obtain equations describing the process in heat exchangers with a boiling heat carrier from (22), (13), and (14) if the boiling fluid is considered a homogeneous medium.

Now, let us examine the simplified model. Let us assume the pipe wall to be absolutely heat conductive, but there is a thermal resistance at its inner boundary which equals the actual plus the resistance of heat conductivity, i.e., the wall has a constant temperature along the radius which equals some real temperature at a definite distance [1]. Then the heat conduction equation (4) with the boundary conditions (5)-(7) is replaced in the system (1)-(7) by the heat balance equation

$$Q_2 = c_m \rho_m \frac{(d_1^2 - d_2^2)}{4d_2} \cdot \frac{\partial \bar{\Phi}}{\partial t} + \beta_2 (\bar{\Phi} - \Theta), \quad (25)$$

where  $\bar{\Phi}$  is the pipe temperature averaged over the radius;  $\beta_2$  is the reduced coefficient of heat exchange taking account of the thermal wall resistance and defined by the formula [1]

$$\beta_2 = \frac{1}{\frac{1}{\alpha_2} + \frac{\sigma d_2}{2\lambda_m} \ln \zeta} = \frac{\alpha_2}{1 + \sigma \text{Bi} \ln \zeta}. \quad (26)$$

Here  $\sigma$  is the fraction of the resistance of heat conduction referred to the inner boundary.

Starting from the assumptions made, in place of  $\alpha_2$  and  $\Phi$  in the energy equation (1) there now enter  $\beta_2$  and  $\bar{\Phi}$ , respectively, i.e.,

$$\frac{1}{v} \cdot \frac{\partial I}{\partial t} + G \frac{\partial I}{\partial l} = \frac{4\beta_2}{d_2} (\bar{\Phi} - \Theta) + \frac{\partial P}{\partial t} - Gv \frac{\partial G}{\partial t}, \quad (27)$$

and (2) and (3) remain as they are.

Going over to the deviations in (25) and (27), replacing the length coordinate by the enthalpy analogously, and taking into account that the heat-exchange coefficient  $\beta_2$  is determined by the expression

$$\beta_2 = \beta_{20} \left\{ 1 + \frac{\kappa}{\mu_q^*} \left[ \frac{1}{\alpha_{20}} \cdot \frac{\partial \alpha_2}{\partial I} i + \mu_g g + \kappa \frac{\partial \alpha_2}{\partial Q_2} \left( \psi - \frac{i}{c_p} - \delta p \right) \right] e^{j\omega t} \right\}, \quad (28)$$

where

$$\kappa = \frac{\beta_{20}}{\alpha_{20}}, \quad \mu_q^* = 1 - \kappa \frac{Q_{20}}{\alpha_{20}} \left( \frac{\partial \alpha_2}{\partial Q_2} \right),$$

we obtain the energy equation

$$\begin{aligned} \frac{di}{dI_0} = & - \left[ \frac{j\omega d_2}{4Q_{20}v_0} + \frac{\kappa\alpha_{20}}{Q_{20}c_p} \cdot \frac{\mu_l j\omega T_M}{(1 + j\omega T_M \mu_q^*)} \right] i - \left[ 1 - \frac{\kappa\mu_g j\omega T_M}{1 + j\omega T_M \mu_q^*} \right. \\ & \left. + \frac{j\omega G_0^2 v_0 d_2}{4Q_{20}} \right] g - \left[ \frac{j\omega T_M \kappa \alpha_{20} \delta}{Q_{20}(1 + j\omega T_M \mu_q^*)} - \frac{j\omega d_2}{4Q_{20}} \right] p + \frac{q}{1 + j\omega T_M \mu_q^*}. \end{aligned} \quad (29)$$

Here

$$T_M = \frac{\rho_m c_m (d_1^2 - d_2^2)}{4\beta_2 d_2}.$$

The continuity and motion equations for the deviations retain their form, i.e., the processes are described by the system (29), (13), and (14) with the boundary conditions (19) upon assuming constancy of the wall temperature along the radius. Therefore, in order for the models with concentrated and distributed wall temperature along the radius to be equivalent, it is necessary to select the heat-exchange coefficient  $\beta_2$  (which means  $\sigma$  also) in such a way that (22) and (29) would turn out to be identical. Comparing the coefficients of  $i$ ,  $g$ ,  $p$ , and  $q$  in these equations, we can arrive at the deduction that two equalities

$$\frac{K}{\mu_q K - \text{Bi}} = \frac{\kappa j\omega T_M}{1 + j\omega T_M \mu_q^*}, \quad (30)$$

$$\frac{2jK \text{Bi}}{y_2^2 (\zeta^2 - 1) (\mu_q K - \text{Bi})} = - \frac{1}{1 + j\omega T_M \mu_q^*}, \quad (31)$$

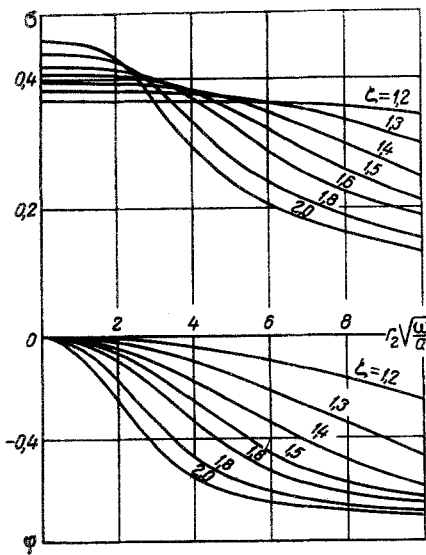


Fig. 1

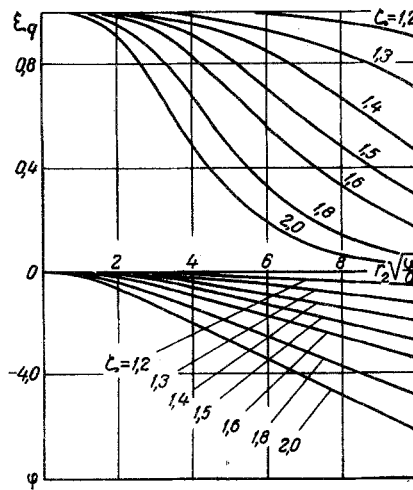


Fig. 2

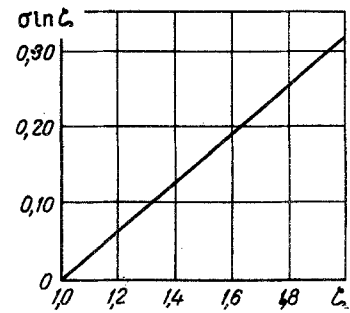


Fig. 3

Fig. 1. Dependence of the fraction of heat conduction resistance referred to the inner pipe boundary on the parameter  $y_2 = r_2 \sqrt{\omega/a}$  for different ratios  $\zeta = d_1/d_2$ .

Fig. 2. Coefficient to convert the perturbations by the external heat flux to an equivalent perturbation of the internal heat source.

Fig. 3. Dependence of the quantity  $\sigma \ln \zeta$  for  $\omega = 0$  on the ratio of the diameters  $\zeta$ .

must be satisfied for the equations to be identical for internal heat liberation in the wall, where the former is obtained from equalizing the coefficients for  $i$ ,  $g$ , and  $p$  and the second from the condition of equality of the coefficients for  $q$ . If the value

$$\omega T_M = \frac{\omega \rho_M c_M (d_1^2 - d_2^2)}{4 \beta_2 d_2} = \frac{r_2 \omega \rho_M c_M (\zeta^2 - 1) a r_2 \alpha_{20}}{2 \beta_2 a r_2 \alpha_{20}} = \frac{y_2^2 (\zeta^2 - 1)}{2 \text{Bi} \kappa}$$

is substituted into (30), then it will be

$$\frac{K}{\mu_q K - \text{Bi}} = \frac{j y_2^2 (\zeta^2 - 1)}{2 \text{Bi} \kappa + j y_2^2 (\zeta^2 - 1) \mu_q^*} \quad (32)$$

Substituting  $\omega T_M$  in (31), we obtain an expression identical to (32). Therefore, if we determine  $\beta_{20}$  from (32) take this value when using the heat balance equation, the result of computing the deviations in the medium parameters will agree with the corresponding solution of the problem taking account of the distributivity of the wall temperature along the radius. Solving this equation for  $\kappa$  we find

$$\kappa = \frac{\beta_{20}}{\alpha_{20}} = \frac{1}{1 + \text{Bi} \left[ \frac{2j}{y_2^2 (\zeta^2 - 1)} - \frac{1}{K} \right]} \quad (33)$$

Now, let us determine the fraction of the thermal resistance  $\sigma$  which must be referred to the inner pipe boundary. Equating the value of  $\kappa$  obtained from (26) to (33), we will have

$$\sigma = \frac{1}{\ln \zeta} \left[ \frac{2j}{y_2^2 (\zeta^2 - 1)} - \frac{1}{K} \right] \quad (34)$$

If the problem of determining  $\beta_{20}$  in the case of heating a heat exchanger outside is examined, then because the flux energy equation in this case is different from the corresponding equation for heating by an inner source only by the presence of a term with a heat flux perturbation  $q$ , we can arrive at the deduction that (34) remains valid for all perturbations ( $i$ ,  $g$ ,  $p$ ). As regards the heat flux perturbations  $q$ , in this case  $\sigma$  should differ from the values obtained in (34). For convenience in the computations, however, it is

convenient to retain the same value of  $T_M$  (and therefore, of  $\sigma$  and  $\beta_{20}$ ) in the term  $q/(1 + j\omega T_M \mu_Q^*)$  for heating from outside as in the remaining terms.

Then to conserve the identity of the equations, the value of  $q$  must be multiplied by the ratio between (23) and (24), i.e., by

$$\xi_q = - \frac{y_2^2 (\zeta^2 - 1) (\mu_q K - Bi) Bi}{y_2 \zeta \sqrt{V} (\mu_q K - Bi) 2 Bi K} = - \frac{i y_2 (\zeta^2 - 1)}{2 \sqrt{K} \zeta} . \quad (35)$$

It follows from (34) and (35), that the quantities  $\sigma$  and  $\xi_q$  are independent of  $Bi$  (i.e., the heat-exchange coefficient), and are determined only by two parameters  $\zeta = d_1/d_2$  and  $y_2 = r_2 \sqrt{\omega/a} = \sqrt{Pd}$ . The values of these quantities were computed on a digital computer in the ranges  $\zeta$  1.2-2 and  $y_2$  0-10. The results of the computation are presented in Figs. 1 and 2.

It is seen from Fig. 1 that, firstly,  $\sigma$  is not a real quantity as is ordinarily assumed, but is complex, which is associated with the finite rate of metal heating, secondly, this quantity is frequency dependent. As it grows the absolute value of  $\sigma$  diminishes since only the metal layers closest to the inner boundary hence succeed in being heated. The graphs in Figs. 1 and 2 afford the possibility of determining the admissibility of using a model with wall temperature concentrated along the radius in every specific case. Thus for heat exchangers with relatively small pipe diameters (the radiation surfaces of locomotive boilers, etc.) the parameter  $y_2$  varies approximately between 0 and 2 (if the cutoff frequency is assumed  $\omega_{\max} \approx 0.2 \text{ sec}^{-1}$ ) and  $\zeta$  between 1.2 and 1.6. As is seen from Figs. 1 and 2, within these limits 1, 2 and  $\sigma$ ,  $\xi_q$  can be assumed real, independent of the frequency, and equal to the values at  $\omega = 0$ . The quantity  $\sigma_{\omega=0}$  varies between 0.364 for  $\zeta = 1.2$  and 0.415 for  $\zeta = 1.6$ , i.e., in this domain of values  $\zeta$  is close to 0.4, which agrees with the results obtained in MoTsKTI [5] and in TsNIIKA by the selection of steam heaters in the computations.

The values of  $\sigma_{\omega=0}$  agree with the quantities computed by the formula recommended in [3]

$$\sigma_{\omega=0} = \frac{1}{\ln \zeta} \left( \frac{\zeta^2}{\zeta^2 - 1} \ln \zeta + \frac{1}{4\zeta^2} - 0.75 \right) \frac{\zeta^2}{\zeta^2 - 1} .$$

However, it is more convenient to use the quantity  $\sigma \ln \zeta$ , whose values are represented in Fig. 3 (for  $\omega = 0$ ) as a function of the ratio between the diameters  $\zeta$  in computing the coefficient  $\sigma \ln \zeta$  by means of (26). It is seen from the figure that this dependence is almost linear, hence, the formula

$$\beta_2 = \frac{\alpha_2}{1 + 0.32(\zeta - 1) Bi}$$

can be recommended for practical computations in the domain  $\zeta = 1-1.6$  for  $y_2$  between 0 and 2.

It should be noted that  $\xi_q \approx 1$  for  $y_2 \approx 0-2$ , i.e., the method of heating (internal heat liberation or heating from outside) exerts practically no influence on  $\sigma$ , hence (36) can be used in computing the dynamics of atomic reactors as well as radiation heating surfaces of boilers with a boiling heat carrier in the case of a strong dependence of the heat carrier properties on the temperature and pressure, or in case they are constant.

As regards computations of the dynamic characteristics of pipelines of great length and large diameters (0.1-0.2 m), then because the range of variation of  $y_2$  is broadened to 0-20 in these cases, it is impossible to assume  $\sigma$  constant even for small  $\zeta$  (see Fig. 1), and either the model with wall temperature distributed along the radius should be used in the computations, or a variable (frequency dependent)  $\sigma$  in the form of a complex quantity should be given when using the concentrated model. This deduction is confirmed by computations performed in the TsNIIKA, which showed that the dynamic characteristics of pipelines in boiler aggregates, computed by means of the concentrated and distributed models (the wall was considered flat), diverge strongly.

#### NOTATION

$r_1, r_2, r$	are the outer, inner, and running radius of the heat-exchanger pipe, respectively, m;
$d_1, d_2$	are the outer and inner diameters, m;
$l$	is the running coordinate of the heat-exchanger length, m;
$t$	is the time, sec;
$v$	is the specific volume of the heat carrier, $\text{m}^3/\text{kg}$ ;

I	is the enthalpy of the heat carrier, J/kg;
$\Theta$	is the temperature of heat carrier, °C;
P	is the pressure of heat carrier, N/m <sup>2</sup> ;
G	is the mass-flow rate of the heat carrier, kg/m <sup>2</sup> · sec;
$c_p$	is the specific heat of the heat carrier, J/kg · deg;
$\rho_M, c_M, \lambda_M$	are the density, specific heat, and heat conduction of the metal pipe, respectively, kg/m <sup>3</sup> , J/kg · deg, J/m · deg · sec;
$a = \lambda_M / \rho_M c_M$	is the temperature conduction of the metal, m <sup>2</sup> /sec;
$\Phi$	is the temperature of the pipe wall, °C;
$\xi$	is the coefficient of hydraulic resistance;
$\gamma$	is the slope of the heat-exchanger pipe;
$Q_1, Q_2$	are the heat flux referred to the outer and inner pipe wall, respectively, W/m <sup>2</sup> ;
$Q_{in}$	is the intensity of internal heat liberation at the wall, W/m <sup>3</sup> ;
$\alpha_2$	is the coefficient of heat exchange from the wall to the heat carrier, W/m <sup>2</sup> · deg;
$j = \sqrt{-1}$	is the imaginary unit;
$\omega$	is the circular frequency, sec <sup>-1</sup> .

### Subscripts

- 0 denotes the static state;  
H denotes the quantity at the initial section of the heat exchanger, i.e., for  $l = 0$ .

### LITERATURE CITED

1. A. A. Armand, in: Heat Exchange at High Heat Loads and Other Special Conditions [in Russian], Gosénergoizdat (1959).
2. E. Gray and G. B. Matthews, Bessel Functions and Their Application in Physics and Mechanics [Russian translation], IL (1953).
3. A. A. Armand and V. V. Krashenninikov, Teploénergetika, No. 1 (1966).
4. V. M. Rushchinskii, Teploénergetika, No. 1 (1967).
5. I. I. Aizenshtat, I. G. Polumordvinova, and E. P. Fel'dman, Method of Computing the Dynamic Characteristics of Steam Heater Sections of Boiler Aggregates [in Russian], No. 15, TsKTI, Moscow (1967).
6. B. N. Seliverstov, Inzh.-Fiz. Zh., 11, No. 4 (1966).